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Christine Ladd-Franklin's Antilogism Revisited

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Abstract

Christine Ladd-Franklin's antilogism—which provides a unified representation of all valid syllogisms—has recently received attention among commentators. While Susan Russinoff argues that Ladd-Franklin delivered one of the most important advances in syllogistic logic in two millennia, Sara Uckelman questions whether Ladd-Franklin did in fact solve the problem Russinoff takes her to address. Here I will focus on the context of the antilogism, namely the algebra of logic tradition, in order to better evaluate these claims. I will particularly focus on the differences in symbolic notation that Ladd-Franklin introduced in contrast to earlier systems. I will then spotlight the (partly hidden) reception of the antilogism in the early 20th century.

Keywords: diagrams, algebra of logic, syllogistic logic.

Introduction

Christine Ladd-Franklin's antilogism for testing syllogistic arguments has recently received a wave of attention in the literature. Starting with Russinoff's paper from 1999 on "The syllogism's final solution" [22], the antilogism has been a topic of discussion, for broadly two distinct reasons. On the one hand, the algebra of logic tradition—which largely ended after Ladd-Franklin's introduction of the concept of the antilogism, since model theory took over as the dominant paradigm, while the algebra of logic tradition was subsequently abandoned and all but forgotten—has been rediscovered by historians of logic, who note the progress Ladd-Franklin's innovations in notation constitute over their predecessors. On the other hand, a growing number of commentators on the history of logic have begun addressing the question of how to better integrate contributions by female logicians into the historiography of the tradition. Women's contributions in the late 19th and early 20th century—already few in numbers during their lifetime—are largely missing from the canon.

Ladd-Franklin's work, in particular her antilogism, an "argument of inconsistency", which presents the single *form*—a notion that Uckelman [26] critically examines—that all ninety-six valid syllogisms are reducible to, and thus a "general characterization of the valid syllogism", is featured in a number of recent contributions to the history of logic. We will see that this reception of Ladd-Franklin's antilogism paradigmatically reflects the issues that arise in trying to reintegrate women's contributions to logic.

The aim of the following three sections is to make a contribution towards a comprehensive reassessment of Ladd-Franklin's antilogism within its historical context. Section 1 examines the recent discussion of the antilogism, particularly engaging with Russinoff's ambitious interpretation and Uckelman's critical response. Section 2 analyzes central innovations in symbolic notation that Ladd-Franklin introduced in her paper "On the algebra of logic" [11, pub. 1883]. Most importantly, we analyze her use of exclusion rather than inclusion as the fundamental logical relation—a notational choice that enabled Ladd-Franklin's formulation of the antilogism as a unified representation of all valid syllogisms. Section 3 traces the substantial but often overlooked reception of the antilogism throughout the early 20th century, documenting its treatment in major logic textbooks and its influence on prominent logicians including C. I. Lewis, Susan Stebbing, and Hans Reichenbach. This will demonstrate that the recent rediscovery of Ladd-Franklin's work represents not the first recognition of its importance, but rather a renewed appreciation of contributions that had significant contemporary impact before being marginalized in the historical record.

1 Recent discussion of the "antilogism"

In "On the algebra of logic" [11, pub. 1883], Ladd-Franklin presents her antilogism in the following form:

> The argument of inconsistency,
>
> $$(a \,\overline{\vee}\, b)(\bar{b} \,\overline{\vee}\, c)(c \vee a)\,\overline{\vee},$$
>
> is therefore the single form to which all the ninety-six valid syllogisms (both universal and particular) may be reduced.

A few lines later, she provides the following proof for this:

> Any given syllogism is immediately reduced to this form by taking the contradictory of the conclusion, and by seeing that universal propositions are expressed with a negative copula and particular propositions with an affirmative copula. [11, p. 40]

Russinoff's "The syllogism's final solution" [22, pub. 1999] can be credited as the first new contribution in decades to point our attention at this important finding in Ladd-Franklin's work. Newer studies of the antilogism equally worth noting are Francine Abeles' "Christine Ladd-Franklin's antilogism" [1, pub. 2023], Frédérique Janssen-Lauret's "Grandmothers of analytic philosophy: The formal and philosophical Logic of Christine Ladd-Franklin and Constance Jones" [10, pub. 2011], and Ahti-Veikko Pietarinen's "Christine Ladd-Franklin's and Victoria Welby's correspondence with Charles Peirce" [19, pub. 2013].

As the title of Russinoff's paper already indicates, she takes Ladd-Franklin to bring syllogistic logic to some kind of "final solution"—a claim I will be critically discussing in what follows, siding with Uckelman's analysis. Russinoff's argues:

> The problem that Aristotle posed and attempted to solve is to give a general characterization of the valid syllogisms. This is the problem that Ladd-Franklin finally solved in the late 19th century. [22, p. 452]

Similarly, Janssen-Lauret's characterization of the contribution that Ladd-Franklin's dissertation made:

> Ladd-Franklin demonstrated that all syllogisms are triads of sentences which share a certain form in her algebraic system, a form such that the truth of the premises is inconsistent with the falsity of the conclusion. [10, p. 7]

Janssen-Lauret then moves on to cite Josiah Royce's enthusiastic praise of the importance of Ladd-Franklin's work:

> Josiah Royce described it as the 'definitive solution of the problem of the reduction of syllogisms' and praised the 'crowning activity in a field worked over since the days of Aristotle .... the work of an American woman. [Royce in 24, p. 60; cf. 10, p. 7]

Janssen-Lauret points out that Ladd-Franklin's solution itself "is not a proof by modern logician's standards", since it uses a "different conception of inconsistency from the present one" [10, p. 7]. According to Janssen-Lauret, we owe the first modern proof of Ladd's theorem to Russinoff's paper:

> Ladd took a triad of statements to be inconsistent just in case the truth of one implied the falsity of one or both of the others. Modern

> logicians hold that a triad of sentences is inconsistent just in case there is no possible interpretation which makes all three statements true. Susan Russinoff has since provided a proof of Ladd's Theorem using contemporary logic. [10, p. 7]

Russinoff shows how Ladd-Franklin's algebra of logic constitutes a variation of the Boole-Peirce-Schröder system. A "revolution in logic" took place in the 19th century, Russinoff claims, "that started with the work by George Boole and Augustus De Morgan" [22, p. 455]. In 1847, both George Boole and Augustus De Morgan published monographs that would set off the algebra of logic tradition, Boole his *Mathematical Analysis of Logic* [3, pub. 1847] and De Morgan his *Formal Logic* [6, pub. 1847]. They argued for an enlargement of syllogistic logic "to include different forms of inference" [22, p. 455].

Boole, in particular, "developed some of the ideas of Leibniz and formulated the first algebraic language for logic". Boole's central insight in *The Mathematical Analysis of Logic* was that "mathematical models are applicable to the study of reasoning in general". He thus "saw a parallel between algebraic operations and logical operations", which he developed in his first algebra of logic [22, p. 455]. Boole's theory was then "revised by De Morgan and Jevons, and then further by C.S. Peirce and Ernst Schröder" [22, p. 455]. Ladd-Franklin's contribution in "On the algebra of logic" then constituted a variation of the Boole-Peirce-Schröder system [22, p. 455].

2 Innovation in symbolic notation in "On the algebra of logic" (1883)

2.1 The classical algebra of logic

While the specific choice in notation varies between different algebras of logic, we largely find the same operations, more specifically, three operations and two relations:

Operations and relations in the classical algebra of logic

Three operations: negation, logical sum, logical product.

- The **negation** of a class a "is represented as $-a$" and "is the class of everything that is in the universe that is not in" a.
- "The **logical sum** of classes a and b is represented as $a + b$ and has as members both the elements of a and the elements of b."

- The **logical product** "of a and b is represented as $a \times b$, or ab and contains those things that are in both a and b" [cf. 22, p. 456].

Two relations: inclusion and equality.

In terms of relations, classical versions of the algebra of logic primarily focus on the inclusion relation. This relation is denoted by the less-than symbol, $a < b$, which indicates that class a is included in class b; in other words, every member of a is also a member of b. This inclusion relation is transitive but not symmetrical, differing from Ladd-Franklin's later notation. The equality of two classes, a and b, is expressed through mutual inclusion, denoted as $a = b$, meaning that both $a < b$ and $b < a$. If two classes are not equal, they are represented as $a \neq b$, as Russinoff [22, p. 456] describes.

We can present the system's *postulates* or *fundamental principles*, where a, b, c, and d are any classes, as follows [22, p. 456]:

Principle of Identity: $a < a$.

Principle of Contradiction: $a \times -a = 0$.

Principle of Excluded Middle: $a + -a = 1$.

Principle of Commutation 1: $ab = ba$.

Principle of Commutation 2: $a + b = b + a$.

Principle of Association 1: $(ab)c = a(bc)$.

Principle of Association 2: $(a + b) + c = a + (b + c)$.

Principle of Distribution: $(a + b)c = ac + bc$.

Principle of Tautology 1: $aa = a$.

Principle of Tautology 2: $a + a = a$.

Principle of Absorption 1: $a + ab = a$.

Principle of Absorption 2: $a(a + b) = a$.

Principle of Simplification 1: $ab < a$.

Principle of Simplification 2: $a < a + b$.

Principle of Composition 1: If $a < b$ and $c < d$, then $a + c < b + d$.

Principle of Composition 2: If $a < b$, and $c < d$, then $ac < bd$.

Principle of the Syllogism: If $a < b$ and $b < c$, then $a < c$.

We can now also express the *categorical statements* by means of the classical algebra of logic, namely as follows[1]:

A	All a's are b.	$a < b$	$a - b = 0$
E	No a's are b.	$a < -b$	$a - b = 0$
I	Some a's are b.	$(a < -b)'$	$ab \neq 0$
O	Some a's are not b.	$(a < b)'$	$a - b \neq 0$

2.2 Exclusion as the basis of the treatment of the syllogisms in "On the algebra of logic"

In contrast to previous conceptions, Ladd-Franklin's treatment of the syllogisms in "On the algebra of logic" is distinguished by using *exclusion* instead of *inclusion* as the fundamental relation. Ladd-Franklin symbolizes exclusion by using the symbol $\overline{\vee}$. The negation of the formula $a \vee b$ is thus $a \overline{\vee} b$. While $a \vee b$ expresses that a is partly b (a is not wholly excluded from b), $a \overline{\vee} b$ symbolizes that a is excluded from b [22, p. 46].

We can now express inclusion and equality in terms of $\vee$ and $\overline{\vee}$, which yields the following expressions for categorical statements in Ladd's version of the algebra of logic:

A	All a is b.	$a \overline{\vee} -b$
E	No a is b.	$a \overline{\vee} b$
I	Some a is b.	$a \vee b$
O	Some a is not b.	$a \vee -b$

Once these relations have been established, we can now introduce the antilogism, Ladd-Franklin's reduction of all forms of valid syllogisms to one (as Russinoff describes it), in its canonical form:

Theorem 2.1 (The Antilogism) $(a \overline{\vee} b)(c \overline{\vee} d) \overline{\vee} (ac \vee b + d)$.

This expressions indicates that:

> the three bracketed formulae form an inconsistent triad. The truth of the first two formulae—no a is b and no c is d—implies the falsity of the final formula, which says that something which is a and c is

[1]The prime symbol ($'$) is not further explained in Russinoff [22, p. 457]. However, given that the two pairs of expressions (**A**, **O** and **E**, **I**) are identical, the prime symbol appears to indicate the different interpretations of the propositions a and b (as indicated by the final column).

> in the union of the classes c and d—that is, something a and c is in the union of the classes c and d—that is, something a and c is either b or d. Ladd notes that the traditional syllogism eliminates a middle term common to two premises. [10, p. 8; cf. 11, p. 33]

Once the notion of the antilogism is introduced, Ladd-Franklin continues by expanding on the consequences of this innovation in notation, developing "an important connection between the study of symbolic logic and syllogistic argument" [18]. As Julia Parker notes, the new notation allowed Ladd-Franklin to make the observation that the central subjects in symbolic logic are "uniting and separating propositions; inserting or omitting terms; and eliminating the least possible amount of content" [18]. Ladd-Franklin thus continues by addressing the question of how "her algebra of logic could be applied to all three subjects".

This still leaves the question unanswered of what exactly Ladd-Franklin was aiming to accomplish by introducing the antilogism. We saw above, that Russinoff takes her to solve a problem that Aristotle posed but failed to solve—as did logicians in the subsequent more than two millennia, namely the problem of giving "a general characterization of the valid syllogism" [22, p. 452]. Sara Uckelman discusses Russinoff's thesis, arguing that her argument is either anachronistic or confusing—after all, we do not even find the notion of a *form* of a syllogism in Aristotle:

> As presented, it was a problem about reducing all forms of syllogism to one form, but the idea of a "form" here is confusing: Syllogisms are typically spoken of as having *figure* and *mood*, not *form*. So this brings us to the initial question: "What is the form of a syllogism?" [26, p. 2]

Uckelman continues her examination of the antilogism arguing that while Ladd-Franklin did solve a problem, "it certainly wasn't Aristotle's, nor had it vexed people for millennia" [26, p. 30], as Russinoff claims. While Aristotle did not have the notion of a "form of a syllogism", he does discuss the notions of a "figure" and a "mood", yet neither of these can be meant either: Ladd-Franklin cannot have meant *figure*, since "it was already well-known that every non-first figure syllogisms can be reduced to a first-figure syllogism" [26, p. 4]. It cannot be *mood* either, since Aristotle was explicit about his lack of interest in "the question 'can all syllogisms be reduced to a single mood'" [26, p. 5]. Uckelman then looks at earlier mentions of the notion of a form, tracing it as it figures in the "hylomorphic distinction between form and matter" since Aristotle (she here refers in particular to Dutilh-Noves' [8] discussion of the form-matter distinction in Alexander of Aphrodisias).

For our purposes here, two conclusions matter. First, Russinoff's thesis is too ambitious. Second, Ladd-Franklin's antilogism ought to be understood in its context: namely in the context of the development of algebras of logic in the late 19th century.

While modern logic textbooks define the inconsistency of three statements in terms of interpretation ("a triad of sentences is inconsistent just in case there is no possible interpretation which makes all three statements true"), Ladd-Franklin gives an alternative account of inconsistency here, namely one that argues that "a triad of statements ... is inconsistent just in case the truth of one implied the falsity of one or both of the others" [10, p. 7]. Overall, there is agreement in the literature that what is distinctive about Ladd-Franklin's version of the algebra of logic is that she gave the algebraic form of syllogistic statements in terms of an *exclusion* rather than an *inclusion* relation, the fundamental relation being symbolized as $a \overline{\vee}\, b$, meaning: a is fully excluded from b.

The central contribution that Ladd-Franklin made in her dissertation is her innovation to Boolean algebra, in the form of a "pair of symmetric copulas". This innovation, in turn, allowed her to "define the 'antilogism' as an 'inconsistent triad' that could be used to represent every valid syllogism" [26, p. 29]. Later on, Ladd-Franklin used the notion of the antilogism in order to refer to a "syllogistic rebuttal", meaning:

> an order-invariant process where "for the usual three statements consisting of *two premises and a conclusion* one substitutes the equivalent *three statements that are together incompatible*". [11, p. 532; 10, p. 8]

As for the advantages of Ladd-Franklin's version of the algebra of logic, two stand out in particular, as Abeles [1] observes. First, the expressed relation is symmetric and intransitive. Second, the quantity of both subject and predicate is universal. Earlier versions of the algebra of logic, such as Boole's and Schröder's, took inclusion to be the fundamental relation, a relation which is transitive and not symmetric. By contrast, Ladd-Franklin based her algebra of logic on the relation of exclusion, which is both symmetric and intransitive. In fact, both $\vee$ and $\overline{\vee}$ are symmetric and intransitive: "$a \vee b$ is defined as a is partly b; $\vee$ and $\overline{\vee}$ are symmetrical and intransitive" [1, p. 191]. In Ladd-Franklin's algebra of logic, the quantity of subject and predicate is furthermore universal. Both $a \vee b$ and $a \overline{\vee}\, b$ can thus be read forward and backward. For an inclusion relation, by contrast, changing it into the corresponding exclusion "requires changing the sign of the predicate" [1, p. 191].

In the final section, we will explore an additional advantage: a small modification allows Ladd-Franklin's notation to express modal statements in a straightforward way—something not possible in algebras of logic that use inclusion as their basic relation.

3 The early 20th century reception of the antilogism

While Ladd-Franklin does not hold the same prominence as Boole, Peirce, Schröder, and others in the algebra of logic tradition, her antilogism remained a significant topic of examination in the 20th century.

In the 1920s, we find, for instance, Reginald Rogers' "The single rule of the antilogism and syllogism" [21, pub. 1920], Eugene Shen's "The Ladd-Franklin formula in logic: The antilogism" [24, pub. 1927], and Ladd-Franklin's response to Shen's paper [12]. After Ladd-Franklin's death in 1930, the reception of her work continued to be substantial. Haskell Curry discussed it in his "A mathematical treatment of the rules of the syllogism" [5, pub. 1936], and so did other major figures, especially in the first half of the 20th century, such as Dotterer in his "A generalization of the antilogism" [7, pub. 1941]. In the second half of the 20th century, the reception of Ladd-Franklin's work petered out, but we still find the antilogism discussed in W.C. Wilcox' "The antilogism extended" [28, pub. 1969], and in Rue Michael Sabre's "Extending the antilogism" [23, pub. 1987].

Furthermore, we find the antilogism mentioned in almost every major logic textbook of the first half of the 20th century, including C. I. Lewis' *Survey of Symbolic Logic* [14, pub. 1918], C. I. Lewis' and C. H. Langford's *Symbolic Logic* [15, pub. 1932], C. A. Mace's *The Principles of Logic: An Introductory Survey* [16, pub. 1934], A. P. Ushenko's *The Theory of Logic: An Introductory Text* [27, pub. 1936], M. R. Cohen's and E. Nagel's *An Introduction to Logic* [4, pub. 1937], and in R. W. Holmes' *Exercises in Reasoning, with an Outline of Logic* [9, pub. 1939]. In addition, there were also Susan Stebbing's *A Modern Introduction to Logic* [25, pub. 1930] and Alice Ambrose and Morris Lazerowitz' *Logic: The Theory of Formal Inference* [2, pub. 1961], whose trreatment of the antilogism I will discuss in a little more detail now.

In Susan Stebbing's work, we find a section on "Reduction and the antilogism", which first discusses how a syllogism in Figures II and III can be reduced to Figure I, "either by conversion of the premises, or by *reductio ad impossibile*". She continues to discuss the method of indirect proof in the context of the syllogism, and shows how:

> the method of reduction can be extended so that an argument stated in a given mood or figure can be expressed in some other mood or some other figure. Any mood can be reduced to any other

> provided that neither mood contains a weakened conclusion or a strengthened premiss. [25, p. 92]

After this exposition of the "traditional theory of indirect reduction" [25, p. 95], Stebbing moves on to the advantages of Ladd-Franklin's method by means of the "inconsistent triad" of the antilogism: "The three propositions, p, q, and $\overline{r}$ cannot be true together", and further,

> When the three propositions contain three and only three terms, they are said by Mrs. Ladd-Franklin to constitute an *Antilogism.*
>
> Given that p, q and $\overline{r}$ form an inconsistent triad, we have the following set of implications:
>
> (1) If p and q, then r.
>
> (2) If p and $\overline{r}$, then $\overline{q}$.
>
> (3) If $\overline{r}$ and q, then $\overline{p}$. [25, p. 95]

Ambrose and Lazerowitz [2, pub. 1961] also discuss the antilogism; in addition, they present a *diagrammatical representation of the antilogism*, namely in the form of a Venn-diagram. In fact, the diagram is a lot more straightforward than it is in the case of evaluating syllogisms by means of the classical algebra of logic notation. They start out with the observation that a "a syllogism is valid if its conclusion *follows* from the premises" [2, p. 71]. From this, we can conclude that the conclusion follows from the premises only if "the negation of the conclusion is inconsistent with the premises". Therefore, "a syllogism is valid only if negating its conclusion results in an inconsistent conjunction of statements" [2, p. 71]. Thus, Argument 1 is valid while Argument 2 is not:

Argument 1.	Argument 2.
All teachers are slave drivers.	All slave drivers are cruel.
All slave drivers are slothful.	All teachers are cruel.
All teachers are slothful.	Some slave drivers are teachers.

In Argument 1, conjoining the premises with the negation of the conclusion yields an inconsistency ("All teachers are slave drivers", "All slave drivers are slothful", and ("All teachers are slothful"), which is logically equivalent to "Some teachers are not slothful". In Argument 2, however, conjoining the premises with the negation of the conclusion yields a "triad of statements which is not inconsistent" [2, p. 71].

Ambrose and Lazerowitz use a diagram in order to demonstrate that the antilogism gives us an easier method for evaluating an argument for its validity, also in its graphical expression: "A simple diagrammatic method for evaluating syllogisms suggests itself immediately", namely one that does not diagram the syllogism itself but instead the "triad of statements obtained by negating the conclusion of the sylogism" [2, p. 72]. We then just need to determine whether the diagram possesses a "contradictory" compartment, meaning one "which is both shaded and has a cross"—if it does, the syllogism is valid, if it does not, it is invalid [2, p. 72]. With this strategy, we obtain the following Venn diagram:

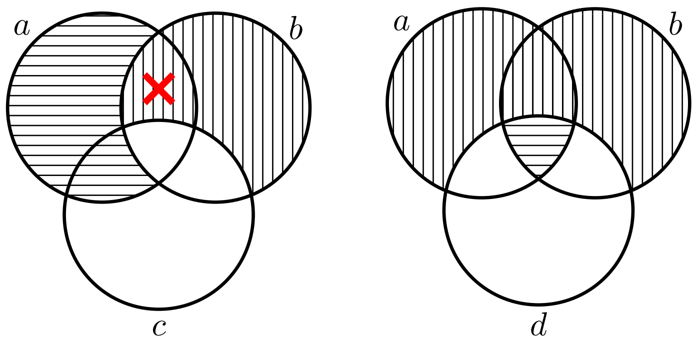

Figure: Adapted from Ambrose and Lazerowitz [2, p. 72, Figure 25].[2]

Ambrose and Lazerowitz conclude that the method of the antilogism:

> is sufficient for testing any syllogism, but it is instructive to see what the conditions are to which a triad of equations and inequations must conform in order to be an inconsistent set, or to be an *antilogism*. [2, p. 72]

All of the examples mentioned—and indeed most other accounts from the first half of the 20th century—understand Ladd-Franklin's antilogism through the lens of an *algebra of classes*. (Russell would shortly after prove that the concept of a class was problematic.) The focus of the literature I cited lies on questions such as what are the necessary and sufficient conditions that a triad of proposition needs to fulfil in order to constitute an antilogism. But we also find later treatments of the antilogism that are not merely concerned with issues pertaining to an algebra of classes.

The antilogism even caught Hans Reichenbach's attention, who argued, in his "The syllogism revised", that "Aristotle does not refer to these forms in his discussion of the syllogism in the Analytics", instead "it took until the 19th century for De Morgan and Ladd-Franklin to point our attention at them"

[2] "The cross is made to span two compartments in order to indicate that one or the other is membered." [2, p. 72]

[20, p. 2, pub. 1952]. Norman and Sylvan, for their part, discuss the nature of the relation that the antilogism expresses when they argue that the antilogism is "quite inaccurately" seen as "merely generalizing contraposition". Instead, they argue that it would be more accurate to say that the antilogism "amalgamates Contraposition and Disjunctive Syllogism" [17, p. 19].

C. I. Lewis, who was already mentioned in the list above, mentions Ladd-Franklin's work in various places. In "Interesting theorems in symbolic logic", Lewis discusses the principle of explosion together with the antilogism:

> Two propositions in the algebra of implication or "calculus of propositions" have been much discussed. They are: "A false proposition implies any proposition," and "A true proposition is implied by any proposition." ...
>
> Any one of these theorems can be proved form the postulates of "Principia Mathematica," from those of Peano, from Schröder's, and from any of the sets given by Huntington. [13, pp. 239, 241]

And he continues with respect to Ladd-Franklin:

> They can also be proved, in somewhat different form, from the assumptions of Mrs Ladd-Franklin's algebra, if the variables of that system are taken to symbolize propositions or propositional functions. [13, p. 241]

Lewis takes "these theorems" to reveal "the divergence of the meaning of 'implies' in the algebra of logic from the 'implies' of valid inference" [13, p. 241].

We find C. I. Lewis praising Ladd-Franklin in various other places, including his *Survey* [14, pp. 108-10] and his *Symbolic Logic* [15, pp. 60ff]. The antilogism would become a logical principle that Lewis and his school understood as a central law of logical reasoning, as well as a controversial issue.

4 Conclusion

We have seen that the recent rediscovery of the antilogism, primarily within efforts to reintegrate women logicians into the canon, is not the first instance of Ladd-Franklin's work being influential. While the *de facto* influence of Ladd-Franklin's antilogism still lacks sufficient appreciation among contemporary commentators, the history of the reception of the antilogism shows its continuous impact on the development of new logical systems in the 20th century.

We can agree with Uckelman that Ladd-Franklin may not have solved a problem posed by Aristotle, or one that "vexed people for millennia" [26, p. 30;

cf. 10]. However, we can still attribute to her a *unique method*. This method first appears in the antilogism and has since sparked a continuous debate about a concept that persists to this day.

I pointed at two distinguishing features of the innovations in notation that Ladd-Franklin introduced, namely, that using the relation of exclusion as the basis of her logical system makes it the case that the symbolic notion of the antilogism is both symmetric and intransitive. The virtues of this notation that I have discussed were mostly matters of convenience and elegance: The antilogism allows us to present inconsistencies more quickly, more elegantly, and it allows us to represent them by simpler diagrammatic means.

The reader might object that these properties alone do not yet establish the superiority of her notational system, which is why I will end on a promissory note, namely with an important consequence of this choice of notation that Roy Cook and I are currently establishing in a joint project: Ladd-Franklin's notation in fact possesses the necessary means for the expression of the statements of modal logic.

We understand $\vee$ and $\overline{\bigtriangledown}$ to express genuine logical possibility and impossibility respectively. This enables us to express all the major axioms of modal logic in terms of her algebra of logic. While both C. I. Lewis—widely credited with developing contemporary modal logic—and Ladd-Franklin were influenced by Hugh MacColl's work, it was Ladd-Franklin who was more directly motivated by considerations related to inconsistency, as well as to the notions of possibility and necessity. These concerns align closely with the foundational ideas of contemporary modal logic, in contrast to the distinct concerns of the relevance logic tradition that later emerged from C. I. Lewis' symbolic notation.

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